Davie Truong

I have read and agree to the collaboration policy. Davie Truong

Homework Heavy

CMPS 102 – Spring 2017 – Homework 3

Solution to problem 2

Reduce the graph to a bipartite graph

Turn Every node in g into two separate nodes, Vin and Vout. The edges connected in g, connects the Vout to the equivalent Vin to maintain similar connectivity. The connecting Vout, Vin edges will be undirected to discover the cycle. Add source S and unit capacity edges from S to each Vin node. Add sink T and unit capacity edges from each Vout to T. The resulting graph g’ can now run Ford-Fulkerson to find max cardinality and a perfect matching.

Proof of Correctness:

Using Ford-Fulkerson to find max cardinality was proved in class. The max cardinality tells us the maximum matching edges between nodes this a max flow subgraph. The edges of that subgraph are perfect if each node appears in exactly one edge. Since each node of the perfect matching subgraph of g’ appears in exactly one edge, the corresponding node in the unmodified graph has one incoming and outgoing edge. Thus, a perfect matching would mean that a cycle cover for the unmodified graph exist. Likewise, if a perfect matching does not occur in the modified graph g’ than we can report that no cycle cover exists.

Time Complexity:

Since we set the edge capacities to 1, finding the perfect matching with the Ford-Fulkerson algorithm would take O(nm) time.

Space Complexity:

O(v+e) to hold the adjacency list.